XLVI. *Problems by* Edward Waring, *M A.* *and* Lucasian *Professor of Mathematics in the University of* Cambridge, *F. R. S.*

PRO.

Read April 21, 1763.

1. INvenire, quot radices impossibiles habet data biquadratica æquatio $x^4+qx^2-rx+s=0$.

1^{mo} Sit $256\,s^3-128\,q^2\,s^2+\overline{144\,r^2\,q+16\,q^4}\times s-27\,r^4-4\,r^2\,q^3$ negativa quantitas, & duas & non plures impossibiles radices habet data æquatio.

2^{do} Sit affirmativa quantitas, & vel $-q$ vel $q^2-4\,s$ negativa quantitas, & datæ æquationis quatuor radices erunt impossibiles.

3^{tio}. Sit nihilo æqualis, & vel $-q$ vel $q^2-4\,s$ negativa quantitas, & datæ æquationis duæ inæquales radicis erunt impossibiles.

2. Invenire, quot radices impossibiles habet data æquatio $x^5+q\,x^3-r\,x^2+s\,x-t=0$.

1^{mo} Si signa terminorum æquationis $w^{10}+10\,q\,w^9+\overline{39\,q^2+10\,s}\times w^8+\overline{80\,q^3+50\,q\,s+25\,r^2}\times w^7+\overline{95\,q^4+124\,q^2\,s-95\,s^2+92\,q\,r^2+200\,r\,t}\times w^6+\overline{66\,q^5-360\,q\,s^2+196\,q^3\,s+118\,q^2\,r^2+260\,r^2\,s+625\,t^2+400\,q\,r\,t}\times w^5+\overline{25\,q^6+40\,s^3-53\,r^4+52\,q^3\,r^2-522\,q^2\,s^2+194\,q^4\,s+708\,q\,r^2\,s+240\,q^2\,r\,t+1750\,q\,t^2-950\,s\,r\,t}\times w^4+\overline{4\,q^7+106\,q^5\,s-80\,q\,s^3-308\,q^3\,s^2-102\,q\,r^4-7\,q^4\,r^2+570\,r^2\,s^2+612\,q^2\,r^2\,s+700\,r^2\,t-3750\,t^2\,s+2500\,t^2\,q^2+80\,r\,t\,q^3-2150\,q\,r\,s\,t}\times w^3+400\,s^4-360\,q^2\,s^3-15\,q^4\,s^2+24\,q^6\,s-8\,q^5\,r^2$

-45

$\overline{-45\, q^2 r^4 - 270\, r^4 s + 140\, r^2 s q^3 + 960\, r^2 s^2 q + 1875}$
$\overline{t^2 r^2 + 1000\, t r s^2 - 5000\, t^2 q s + 1750\, t^2 q^3 + 40\, t r q^4}$
$\overline{+ 600\, t r^3 q - 1650\, t r s q^2} \times w^2 + \overline{36\, q^5 s^2 - 224\, q^3 s^3}$
$\overline{+ 320\, q s^4 + 4\, q^3 r^4 + 27\, r^6 - 40\, r^2 s^2 + 434\, r^2 q^2 s^2 -}$
$\overline{24\, r^2 s q^4 - 198\, r^4 q s + 5000\, t^2 s^2 - 450\, t r^3 s - 6250}$
$\overline{t^3 r + 675\, t^2 q^4 - 3750\, t^2 q^2 s + 3000\, t^2 r^2 q + 60\, t r^3 q^2}$
$\overline{+ 200\, t r s^2 q - 330\, t r q^3 s} \times w + 3125\, t^4 - \overline{3750\, q r t^3}$
$\overline{+ 2000\, s^2 q + 2250\, r^2 s - 900\, s q^3 + 825\, r^2 q^2 + 108\, q^5}$
$\overline{\times t^2 - 1600\, s^3 r - 560\, r q^2 s^2 - 16\, r^3 q^3 + 630\, r^3 q s +}$
$\overline{72\, r s q^4 - 108\, r^5} \times t + 256\, s^5 - 128\, q^2 s^4 + 144\, r^2 q s^3$
$+ 16\, q^4 s^3 - 27\, r^4 s^2 - 4\, r^2 q^3 s^2 = 0$. continuo mutentur de + in —; & — in +; nullas impossibiles radices habet data æquatio.

2do. Si signa terminorum æquationis haud continuo mutentur de + in — & — in +; duæ vel quatuor datæ æquationis radices erunt impossibiles, prout ultimus ejus terminus sit negativa vel affirmativa quantitas.

3tio. Si ultimus ejus terminus nihilo sit æqualis, & signa terminorum æquationis haud continuo mutentur de + in — & — in +; tum vel quatuor vel duæ radices datæ æquationis erunt impossibiles, prout duo & non plures ultimi datæ æquationis termini nihilo sint æquales, necne.

P R O.

Sint x, y, v, abscissa, ordinata & area datæ curvæ, & sit $y^n + \overline{a + bx} \times y^{n-1} + \overline{c + dx + ex^2} \times y^{n-2} + \overline{f + gx + hx^2 + kx^3} \times y^{n-3} +$ &c. $= 0$: invenire, utrum area (v) quadrari potest, necne.

Supponamus æquationem ad aream esse $v^n + \overline{A + Bx + Cx^2}\, v^{n-1} + \overline{D + Ex + Fx^2 + Gx^3 + Hx^4} \times$

$v^{n-2} + \overline{I + Kx + Lx^2 + Mx^3 + Nx^4 + Ox^5 + Px^6} \times v^{n-3} +$ &c. $= 0$. & consequenter erit $nyv^{n-1} + \overline{n-1}$

$$\left.\begin{array}{l} \overline{A + Bx + Cx^2}\, y v^{n-2} + \overline{n-2} \times \overline{D \times Ex + Fx^2 + Gx^3 + Hx^4} \\ \quad \overline{B + 2Cx}\; v^{n-1} + \quad \overline{E + 2Fx + 3Gx^2 + 4Hx^3} \\ \times\, y v^{n-3} + \text{ \&c.} \\ \times\, v^{n-2} + \text{ \&c.} \end{array}\right\} = 0.$$

Ex quibus æquationibus, si methodis notis extermi-netur (v), habebimus æquationem, quæ exprimit relationem inter (x) & (y). Hujus autem æquationis coefficientes æquari debent coefficientibus datæ æquationis $y^n + \overline{a + bx}\, y^{n-1} + \overline{c + dx + ex^2} + y^{n-2} +$ &c. $= 0$; & si quantitates A, B, C, &c. exinde determinari possunt, curva quadratur, est enim $v^n + \overline{A + Bx + Cx^2} \times v^{n-1} + \overline{D + Ex + Fx^2 + Gx^3 + Hx^4} \times v^{n-2} +$ &c. $= 0$; aliter autem quadrari non potest.

Ex. Sit data æquatio $y^2 + x^2 - 1 = 0$, & supponamus æquationem ad aream $v^2 + D + Ex + Fx^2 + Gx^3 + Hx^4 = 0$; & erit $2vy + E + 2Fx + 3Gx^2 + 4Hx^3 = 0$, ita reducantur hæ duæ æquationes in unam, ut exterminatur (v), & resultat æquatio $y^2 + \dfrac{16H^2x^6 + 24HGx^5 + \overline{16HF + 9G^2}\,x^4 + \overline{8EH + 12FG}\,x^3 + \overline{6GE + 4F^2}\,x^2 + 4FEx + E^2}{4 \times Hx^4 + Gx^3 + Fx^2 + Ex + D} = 0$; debet autem fractio $\dfrac{16H^2x^6 + 24HGx^5 + \overline{16HF + 9G^2}\,x^4 + \overline{8EH + 12FG}\,x^3 + \overline{6GE + 4F^2}\,x^2 + 4FEx + E^2}{4 \times Hx^4 + Gx^3 + Fx^2 + Ex + D}$ esse $x^2 - 1$; & consequenter

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\begin{aligned}
4H &= 16H^2 \\
4G &= 24HG \\
4F - 4H &= 16HF + 9G^2 \\
4E - 4G &= 8HE + 12FG \\
4D - 4F &= 6GE + 4F^2 \\
-4E &= 4FE \\
-4D &= E^2
\end{aligned}
$$

ſed e methodo communes diviſores inveniendi conſtat has æquationes inter ſe contradictorias eſſe, & conſequenter curvam haud generaliter eſſe quadrabilem.

THEO.

Sint x, y, v, abſciſſa & ordinatæ curvarum ABCDEFGHI &c. & $A\beta\gamma\delta\varepsilon$ &c. & ſit $y = px^n$, & $v =$

$$
\frac{n}{2.3} pa^{n-1}x - \frac{n\times\overline{n-1}\times\overline{n-2}}{30\times 2\times 3} pa^{n-3}x^3 + \frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}}{42\times 2\times 3\times 4\times 5} pa^{n-5}x^5 - \frac{n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}\times\overline{n-5}\times\overline{n-6}}{30\times 2\times 3\times 4\times 5\times 6\times 7} pa^{n-7}x^7 + \frac{5n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}\times\overline{n-5}\times\overline{n-6}\times\overline{n-7}\times\overline{n-8}}{66\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9} pa^{n-9}x^9 - \frac{691\times n\times\overline{n-1}\times\overline{n-2}\times\overline{n-3}\times\overline{n-4}\times\overline{n-5}\times\overline{n-6}\times\overline{n-7}\times\overline{n-8}\times\overline{n-9}\times\overline{n-10}}{2730\times 2\times 3\times 4\times 5\times 6\times 7\times 8\times 9\times 10\times 11} pa^{n-11}
$$

$x^{11} +$ &c. cujus ultimus terminus debet eſſe x^{n-1} vel x^{n-2}, prout (n) eſt par vel impar numerus.

Sit $x = AP = a$, biſecetur AP in T in duas æquales partes, & ducatur linea ETδ, & ſi AE, EM, AM, jungantur; erit triangulum AEM $=$ TP$\varepsilon\delta$T areæ.

Deinde,

Deinde, bisecentur T P, A T in R and V, & ducantur R G, C V γ; & jungantur A C, C E, E G, G M; & erunt duo triangula A C E + E G M = V T $\delta\gamma$ V areæ.

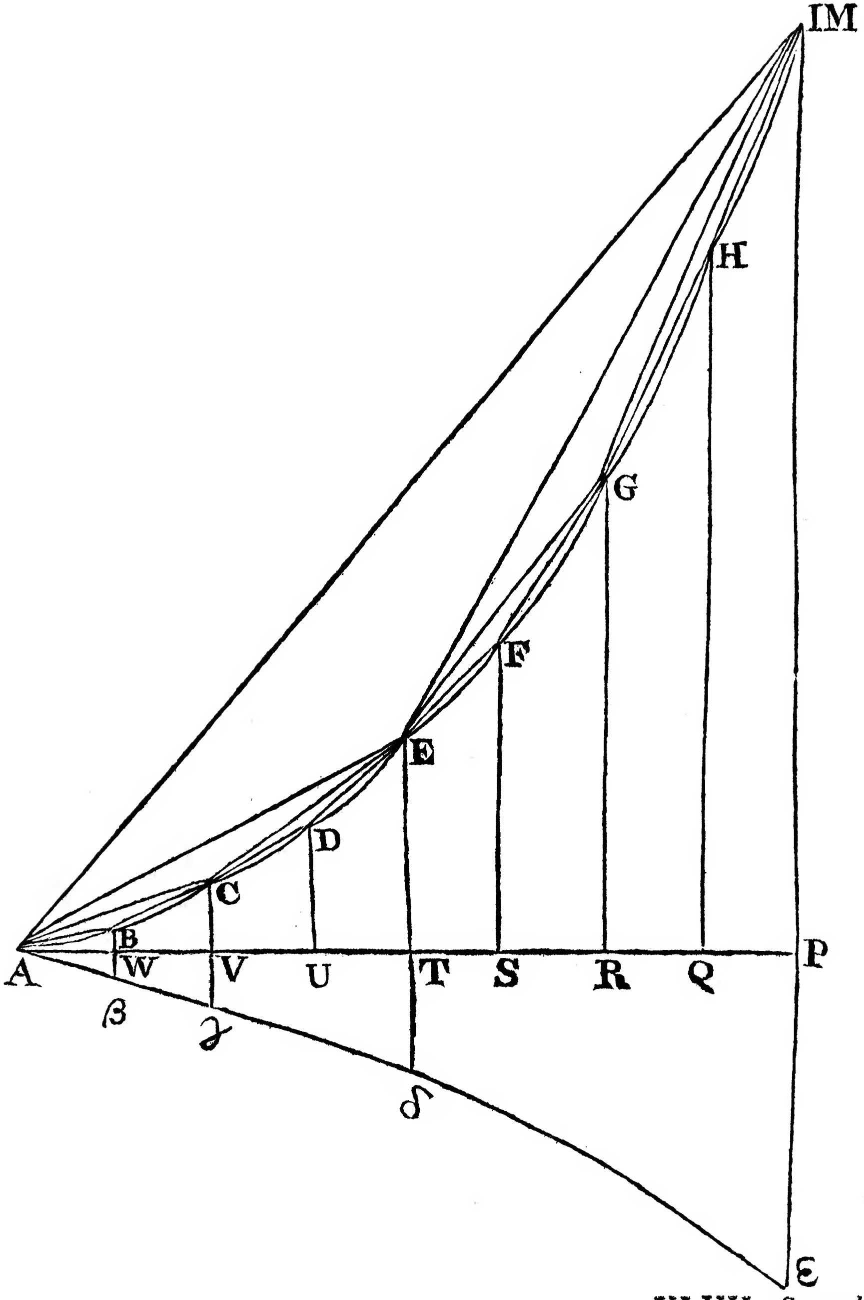
Eodem modo, si partes A V, V T, T R, R P iterum bisecentur in W, U, S, Q, & ducantur lineæ B W β, U D, S F, Q H; & jungantur A B, B C, C D, D E, E F, F G, G H, H M; erunt quatuor triangula A B C + C D E + E F G + G H M = W V $\gamma\beta$ W areæ; & sic deinceps.

Cor. 1. Si curva A B C & M sit conica parabola, $(c, e)\ y = pix^2$, erit $v = \frac{1}{3} pax$; & A $\beta\gamma\delta$ &c. erit recta linea; & propositio eadem est cum notissimâ propositione Archimedis de quadraturâ parabolæ.

Cor. 2. Si $y = px^3$, erit $v = \frac{1}{2} pa^2 x$, & A $\beta\gamma\delta$ &c. iterum recta linea.

Cor. 3. Datâ curvâ, cujus æquatio est $y = px^{2n}$, inveniri potest altera curva, cujus dimensiones sunt $(2n-1)$, in quâ summæ triangulorum ad singulas bisectiones erunt respectivè æquales summis triangulorum datæ curvæ.

His adjici potest, quod si loco bisectionis abscissa A P aliâ quâvis ratione in æquales partes dividatur, summæ triangulorum curvæ A B C D &c. ad singulas divisiones æquales erunt segmentis curvæ A $\beta\gamma\delta$ &c.



XLVII. *Second*